

Correlation Functions of Massless Interacting Scalar Fields in de Sitter Space

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Abstract

We examine the behavior of correlation functions for a massless scalar field in de Sitter space with a quartic interaction. We find that two-loop corrections are relevant, and the resummation of these corrections generates a complicated structure whereby high momentum modes stay massless while low momentum modes develop a dynamical mass.

I. INTRODUCTION

The goal of this paper is to address the following question: what do the correlation functions look like for a massless field in de Sitter space with a ϕ^4 interaction?

This question is of more than merely formal interest; it has direct connections to issues in inflationary theory. The inflationary geometry has many similarities to de Sitter space (especially in the earlier stages of inflation). Furthermore, scalar fields play a fundamental role in inflation [1], and correlation functions of these fields are recorded in the cosmic microwave background.

Current observations of the CMB from the WMAP experiment [2] are currently almost exactly consistent with a scale invariant power spectrum. However, it would be surprising if the inflaton had no interactions whatsoever, and such interactions are expected to produce deviations from exact scale invariance [3]. Several experiments have been searching for such nongaussianities. For comparison of these observations to theory, it is important to be able to calculate correlation functions of these scalar fields in the inflationary background.

Unfortunately, the scalar fields in inflation are typically light ($m \ll H$), and for these small masses, it turns out that the loop corrections are important. In fact, loop corrections involving massless fields are infrared divergent [4–6]. To compare theoretical predictions to observations, it is crucial to be able to treat these divergences rigorously. This has led to a lot of work using many different approaches to try and resum these divergences formally both for scalars and gravitons [7–44]. In some approaches the fact that one loop corrections are divergent for a massless field results in a self-consistent mass being developed. In other approaches, the propagator becomes time dependent, breaking de Sitter invariance, and in yet other approaches particle production leads again to an induced mass for the field. All these techniques qualitatively lead to the result that the massless fields pick up a nonperturbative mass of order $\sqrt{\lambda}H$ once interactions are included.

However, one would expect that a short wavelength mode of the scalar field should be completely unaffected by the de Sitter curvature (in much the same way as current lab experiments do not take into account the curvature of the universe). A long wavelength mode, on the other hand, can plausibly be affected by the curvature and may pick up a dynamical mass (indeed, this *must* happen in order to prevent divergences from occurring.). In short, we expect any corrections to be momentum dependent, with large effects only for long wavelength modes. However, the loop effects found in the various approaches above often seem to affect all wavelengths equally, and usually the short wavelength modes acquire a mass.

The reason for this is that the analyses in most of the above papers typically focus on *one-loop* corrections. However, in ϕ^4 theory, the one-loop corrections have a very non-generic feature; they are *independent* of the external momenta. This leads to the surprising effect detailed in the previous paragraph. This feature suggests that the one-loop corrections may not encode the relevant physics; indeed, all one-loop corrections in ϕ^4 theory can be completely canceled by the addition of a mass counterterm.

This further suggests that the resolution of the divergences involves a study of the two-loop corrections in this theory. Unlike the one-loop corrections, the 2-loop corrections are not independent of the external momenta. The long wavelength modes will then receive mass corrections which are different from the corrections to the short distance modes; this then allows us to cancel off the mass for the short distance modes using the counterterm, while maintaining a nonzero mode for the long distance modes which can cancel the IR divergence.

Our focus in this paper is to analyze in detail how these 2-loop corrections affect the correlation functions. Unsurprisingly, they are much more computationally difficult than the one-loop corrections, and so we will be unable to find closed form analytic results. However, many qualitative features can be extracted. In particular, we show that one loop corrections are indeed canceled by counterterms, while two-loop corrections produce a momentum dependent mass which is zero for the short distance modes. Hence the long wavelength modes acquire a dynamical mass, while the short wavelength modes remain massless, as expected.

To see this effect, which is nonperturbative, we will need to resum the two loop corrections. Technically, we resum a subset of the two-loop diagrams which are expected to produce the leading infrared divergences. This resummation will be found to produce a momentum dependent effective mass. The scaling of the masses turns out to be very

different from previous calculations in the literature.

As we shall discuss below, the difference between our results and the results in the literature appears to hinge on a different definition of the term 'massless scalar' (most importantly, reference [41] employs techniques extremely similar to this paper, but with a different definition of the massless limit). To define what we mean by this term, we use the general feature that the de Sitter propagator, on scales smaller than the Hubble scale, should locally resemble a Minkowski space propagator. In our definition, a massless field is one which for shorter length scales (i.e. distances parametrically smaller than the Hubble scale) has a propagator similar to a massless Minkowski space propagator. This is not the only possible choice (reflecting the difference with other definitions in the literature), but is a well defined one. The more naive definition would have been that the propagator should approach the propagator for a massless field at low momenta, but this appears to run into divergences.

II. REVIEW OF THE IN-IN FORMALISM

We begin by reviewing the in-in formalism for ϕ^4 theory. This section deals with the free field. These rules have already been derived and presented elsewhere (e.g. [4, 21, 30]); we refer the interested reader to these papers for further details.

We will take the metric of de Sitter space to be

$$ds^2 = \frac{1}{H^2\tau^2} \left(d\tau^2 - \sum_{i=1}^3 dx_i^2 \right) \quad (1)$$

We will also consider a scalar field propagating in this geometry; the field will have mass m and a quartic interaction. The Lagrangian is then

$$\mathcal{L}(\phi) = \sqrt{g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 \right] \quad (2)$$

We note that we will be focusing on very light fields. To leading order, therefore, we will set $m^2 = 0$. Furthermore, we will have to work in the in-in formalism. In this formalism, the number of fields is doubled; for the scalar field, we have the fields ϕ_+, ϕ_- . We additionally define

$$\phi_C = \frac{1}{2}(\phi_+ + \phi_-) \quad \phi_\Delta = \phi_+ - \phi_- \quad (3)$$

The in-in Lagrangian is then defined as $\mathcal{L} = \mathcal{L}(\phi_+) - \mathcal{L}(\phi_-)$.

There are four propagators corresponding to the two sets of scalar fields. The Keldysh propagator is denoted F and is defined as

$$iF(x, y) = \langle \phi_C(x) \phi_C(y) \rangle \quad (4)$$

In addition, we have the advanced and retarded propagators

$$G^R(x, y) = \langle \phi_C(x) \phi_\Delta(y) \rangle = i\theta(x^0 - y^0)(\langle \phi(x) \phi(y) \rangle - \langle \phi(y) \phi(x) \rangle) \quad (5)$$

$$G^A(x, y) = G^R(y, x) \quad (6)$$

The fourth propagator $\langle \phi_\Delta(x) \phi_\Delta(y) \rangle$ is identically zero.

In Feynman diagrams, ϕ_C is denoted by a solid line, and ϕ_Δ by a dashed line. The propagators (after a Fourier transform) are denoted by

$$\overline{\tau_1} \text{-----} \tau_2 = F(k, \tau_1, \tau_2) \quad (7)$$

$$\overline{\tau_1} \text{-----} \tau_2 = -iG^R(k, \tau_1, \tau_2) \quad (8)$$

There are two vertices in ϕ^4 theory.

$$\begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} = -i\frac{\lambda}{4}a^4(\tau) \quad (9)$$

$$\begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} = -i\lambda a^4(\tau) \quad (10)$$

We can calculate these propagators in the free field limit i.e. when $\lambda = 0$. For free fields, the propagators can be found to be (we will denote the free field limit by a 0 superscript)[21]

$$F_{m^2}^{(0)}(k, \tau_1, \tau_2) = \frac{\pi H^2}{4}(\tau_1 \tau_2)^{3/2} \text{Re}(H_\nu(-k\tau_1)H_\nu^*(-k\tau_2)) \quad (11)$$

$$G_{m^2}^{R(0)}(k, \tau_1, \tau_2) = -\theta(\tau_1 - \tau_2) \frac{\pi H^2}{2}(\tau_1 \tau_2)^{3/2} \text{Im}(H_\nu(-k\tau_1)H_\nu^*(-k\tau_2)) \quad (12)$$

where $\nu^2 = \frac{9}{4} - \frac{m^2}{H^2}$.

There are several limits of the free field F -propagator that are of interest. The first is the zero mass limit. For zero mass, we have

$$F_{m^2=0}^{(0)}(k, \tau_1, \tau_2) = \frac{H^2}{2k^3}[(1 + k^2\tau_1\tau_2)\cos(k(\tau_1 - \tau_2)) + k(\tau_1 - \tau_2)\sin(k(\tau_1 - \tau_2))] \quad (13)$$

$$G_{m^2=0}^{R(0)}(k, \tau_1, \tau_2) = \theta(\tau_1 - \tau_2) \frac{H^2}{k^3}[(1 + k^2\tau_1\tau_2)\sin(k(\tau_1 - \tau_2)) - k(\tau_1 - \tau_2)\cos(k(\tau_1 - \tau_2))] \quad (14)$$

For small k the Keldysh propagator goes as $\frac{1}{k^3}$

$$F_{m^2=0}^{(0)}(k, \tau_1, \tau_2) \sim \frac{H^2}{2k^3} \quad (15)$$

This leads to divergences in loop diagrams. For finite masses, the low momentum behavior of the propagator is expressible in terms of $\frac{m^2}{3H^2} = \epsilon \ll 1$ and is found to be

$$F_{m^2}^{(0)}(k, \tau_1, \tau_2) \sim \frac{H^2}{2k^3}(k^2\tau_1\tau_2)^\epsilon \quad (16)$$

which regulates the infrared divergences found at $m^2 = 0$.

III. ONE-LOOP CORRECTIONS

We first briefly consider the one-loop corrections to the massless theory. The relevant diagrams are shown below. The third and fourth diagrams represent counterterms.



FIG. 1: One loop corrections

In a perturbation expansion, the first diagram is of the form

$$-i\frac{\lambda}{4}a^4(\tau_3)G^{(0)}(k, \tau_1, \tau_3) \left[\int d^3q F^{(0)}(q, \tau_3, \tau_3) \right] F^{(0)}(k, \tau_3, \tau_2) \quad (17)$$

For $m^2 = 0$, $F^{(0)}(q, \tau_3, \tau_3) \sim \frac{1}{q^3}$ at small q , and so the integral is divergent. The massless theory therefore does not seem to have a good perturbation expansion. However, the infinite correction can be canceled by a suitable choice of the counterterms. Since the loop diagram is independent of external momenta, canceling the mass at high energies also cancels the mass at zero momentum. This will again result in a massless zero mode which will lead to IR divergences. We must therefore go beyond one loop to see whether the divergences can be resolved.

IV. TWO-LOOP CORRECTIONS

The two loop correction has the generic form of a sunrise diagram. There are in fact three separate two-loop corrections which occur in this theory, which we denote $\Sigma_{1,2,3}$.

The first set of corrections is shown in Figure 2. These corrections can be written explicitly as



FIG. 2: Corrections contributing to Σ_1

$$\begin{aligned} \Sigma_1(k, \tau_1, \tau_2) = & i\lambda^2 a^4(\tau_1) a^4(\tau_2) \int d^3 k_1 \int d^3 k_2 F(k_1, \tau_1, \tau_2) F(k_2, \tau_1, \tau_2) G^R(k - k_1 - k_2, \tau_1, \tau_2) \\ & - i\frac{\lambda^2}{4} a^4(\tau_1) a^4(\tau_2) \int d^3 k_1 \int d^3 k_2 G^R(k_1, \tau_1, \tau_2) G^R(k_2, \tau_1, \tau_2) G^R(k - k_1 - k_2, \tau_1, \tau_2) \end{aligned} \quad (18)$$

This loop can be thought of as inducing a new vertex corresponding to a (nonlocal) term in the Lagrangian proportional to $\phi_C \phi_\delta$.

There is an analogous vertex where we reverse the time direction; this diagram is the mirror image of the diagram above. This mirrored vertex will be denoted Σ_2 and can be thought of as inducing a new vertex corresponding to a (nonlocal) term in the Lagrangian proportional to $\phi_\delta \phi_C$.

Finally, there is a third set of loop corrections which induce a new vertex corresponding to a (nonlocal) term in the Lagrangian proportional to $\phi_\Delta \phi_\Delta$. This set of diagrams will be denoted Σ_3 , and can be written as

$$\Sigma_3(k, \tau_1, \tau_2) = \frac{\lambda^2}{4} a^4(\tau_1) a^4(\tau_2) \int d^3 k_1 \int d^3 k_2 G^R(k_1, \tau_1, \tau_2) G^R(k_2, \tau_1, \tau_2) F(k - k_1 - k_2, \tau_1, \tau_2) \quad (19)$$

$$+ \frac{\lambda^2}{4} a^4(\tau_1) a^4(\tau_2) \int d^3 k_1 \int d^3 k_2 G^A(k_1, \tau_1, \tau_2) G^A(k_2, \tau_1, \tau_2) F(k - k_1 - k_2, \tau_1, \tau_2) \quad (20)$$

$$- \lambda^2 a^4(\tau_1) a^4(\tau_2) \int d^3 k_1 \int d^3 k_2 F(k_1, \tau_1, \tau_2) F(k_2, \tau_1, \tau_2) F(k - k_1 - k_2, \tau_1, \tau_2) \quad (21)$$

For compactness, we shall replace all loops Σ_i in Feynman diagrams by a insertion labeled with the appropriate index i . Therefore for example, we shall indicate a Σ_3 insertion by a dot with a 3 subscript as shown in figure 3.



FIG. 3: A condensed notation for the loop integrals

V. THE HIGH MOMENTUM REGION

These diagrams are difficult to compute exactly. We are, however, interested in the most divergent pieces of these diagrams; this will allow us to make several approximations to obtain an estimate of these loop corrections. We will focus on Σ_1 at the beginning, and apply the same approximations to the other integrals afterward.

We first note that the IR divergences primarily come from regions of the loop integral where one of the F propagators has a small momentum flowing through it. Indeed, if we were to replace F by $F^{(0)}$ in the above integrals, they would all be infrared divergent because $F^{(0)}(k, \tau_1, \tau_2) \sim \frac{1}{k^3}$. This IR divergence is presumably regulated when the full F propagator is used. We therefore conclude that the low momentum behavior of the full F propagator is significantly different from the tree level propagator F^0 . This modification by the higher order corrections will moderate the infrared behavior and make the integrals finite. Now, although the F propagator is modified in such a way that the integrals are finite, the divergences must reappear if we take the interaction $\lambda \rightarrow 0$. It must therefore be the case that we get a large contribution (i.e. parametrically larger as a function of λ) from the region of integration where the momenta flowing through an F propagator is small. This further indicates that we can keep only the terms with the largest number of F propagators, which are expected to have the largest contributions.

On the other hand, the G propagator does not have infrared divergences when the momentum flowing through it is small. This means that there is no requirement for the corrections to modify it significantly at low momenta. We will assume the corrections to G are small (i.e. suppressed by powers of λ) which will be confirmed by later calculations. We will therefore (to leading order) replace G by G^0 in the above expressions.

Furthermore, the integration over momenta is expected to be dominated by the regions when both F propagators have small momenta in their arguments. In particular, we would expect the dominant part of the integral for Σ_1 to come from the integration region where both internal momenta k_1, k_2 are much smaller than the external momentum k i.e. $k_1, k_2 \ll k$. The integral then simplifies to

$$\Sigma_1(k, \tau_1, \tau_2) \simeq ia^4(\tau_1)a^4(\tau_2)G^{R(0)}(k, \tau_1, \tau_2) \times [c(k, \tau_1, \tau_2)]^2 \quad (22)$$

where we have defined

$$c(k, \tau_1, \tau_2) = \lambda \int_0^k d^3k_1 F(k_1, \tau_1, \tau_2) \quad (23)$$

The next point to note is that the leading term in the $c(k, \tau_1, \tau_2)$ integral above is expected to be time independent. The integral diverges if F is replaced by the tree level F^0 propagator, which for small k leads to an integral of the form $\frac{d^3k}{k^3}$. The divergent term is time independent. When the divergence is regulated, the integral will be large but finite; it is reasonable to expect that the leading value (in an expansion in λ) will continue to be time independent. We will defined this leading term to be $m_1^2(k)$.

With these approximations, we are able to simplify the Σ_1 integral to

$$\Sigma_1(k, \tau_1, \tau_2) \simeq ia^4(\tau_1)a^4(\tau_2)m_1^4(k)G^{R(0)}(k, \tau_1, \tau_2) \quad (24)$$

We can now resum the two loop corrections. For the retarded propagator, the corrections that we wish to sum are of the form shown in Figure 4.

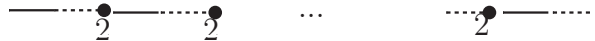


FIG. 4: Corrections to the retarded propagator

These lead to an equation for the full propagator

$$G^R(k, \tau_1, \tau_2) = G^{R(0)}(k, \tau_1, \tau_2) \quad (25)$$

$$+ m_1^4(k) \int \frac{d\tau_3}{(H\tau_3)^4} \frac{d\tau_4}{(H\tau_4)^4} G^{R(0)}(k, \tau_1, \tau_3) G^{R(0)}(k, \tau_3, \tau_4) G^{R(0)}(k, \tau_4, \tau_2) \quad (26)$$

$$+ m_1^8(k) \int \frac{d\tau_3}{(H\tau_3)^4} \frac{d\tau_4}{(H\tau_4)^4} \frac{d\tau_5}{(H\tau_5)^4} \frac{d\tau_6}{(H\tau_6)^4} G^{R(0)}(k, \tau_1, \tau_3) G^{R(0)}(k, \tau_3, \tau_4) G^{R(0)}(k, \tau_4, \tau_5) \quad (27)$$

$$\times G^{R(0)}(k, \tau_5, \tau_6) G^{R(0)}(k, \tau_6, \tau_2) \quad (28)$$

$$+ \dots \quad (29)$$

We can perform the sum by comparing it to the expansion when a mass term is treated as a perturbation. A massive field has the exact retarded propagator $G_{m^2}(k, \tau_1, \tau_2)$. On the other hand, in the same theory, the mass term $-m^2\phi_C\phi_\Delta$ can be treated as a perturbation to the massless theory. We can match the exact solution to the perturbation expansion; this yields

$$G_{m^2}^{R(0)}(k, \tau_1, \tau_2) = G_{m^2=0}^{R(0)}(k, \tau_1, \tau_2) \quad (30)$$

$$- m^2 \int \frac{d\tau_3}{(H\tau_3)^4} G_{m^2=0}^{R(0)}(k, \tau_1, \tau_3) G_{m^2=0}^{R(0)}(k, \tau_3, \tau_2) \quad (31)$$

$$+ m^4 \int \frac{d\tau_3}{(H\tau_3)^4} \frac{d\tau_4}{(H\tau_4)^4} G_{m^2=0}^{R(0)}(k, \tau_1, \tau_3) G_{m^2=0}^{R(0)}(k, \tau_3, \tau_4) G_{m^2=0}^{R(0)}(k, \tau_4, \tau_2) \quad (32)$$

$$+ \dots \quad (33)$$

Comparing this to the previous expansion involving Σ_1 , we find

$$G^R(k, \tau_1, \tau_2) = \frac{1}{2}(G_{m_1^2(k)}^{R(0)}(k, \tau_1, \tau_2) + G_{-m_1^2(k)}^{R(0)}(k, \tau_1, \tau_2)) \quad (34)$$



FIG. 5: Corrections to the Keldysh propagator

Similarly, the F propagator receives contributions of the form shown in Figure 5. Just like the the retarded propagator, we find that the contribution of these terms is $\frac{1}{2}(F_{m_1^2(k)}(k, \tau_1, \tau_2) + F_{-m_1^2(k)}(k, \tau_1, \tau_2))$.

We also need to consider contributions which involve a Σ_3 vertex insertion. These are all of the form shown in Fig. 6.

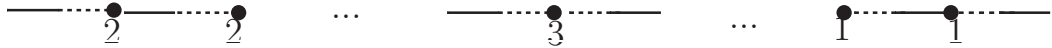


FIG. 6: Other corrections to the Keldysh propagator

Comparing to the expansion for the retarded propagator, we find that these contributions can be written as

$$\int \frac{d\tau_3}{(H\tau_3)^4} \frac{d\tau_4}{(H\tau_4)^4} G^R(k, \tau_1, \tau_3) \Sigma_3(k, \tau_3, \tau_4) G^A(k, \tau_4, \tau_2) \quad (35)$$

We shall treat these diagrams in perturbation theory; that is, we shall not include them in the resummation. We will hence ignore these terms for the remainder of the discussion. It would be interesting to examine the effects of these diagrams; we shall leave this for future work.

Summarizing our results so far, we have found that the resummed Keldysh propagator is of the form

$$F(k, \tau_1, \tau_2) = \frac{1}{2}(F_{m_1^2(k)}(k, \tau_1, \tau_2) + F_{-m_1^2(k)}(k, \tau_1, \tau_2)) \quad (36)$$

where $m_1^2(k)$ is the leading term in the integral (23). Note that this is *not* the propagator for a massive field.

VI. THE LOW MOMENTUM REGION

Unfortunately, our result above cannot hold for all external momenta. For if it did, (36) would be the exact solution for the propagator. But now the term in the propagator $F_{-m_1^2(k)}(k, \tau_1, \tau_2)$ grows in the infrared even faster than $F_0(k, \tau_1, \tau_2)$. From (23), we would find $m_1^2(k)$ to be divergent. This implies that the solution is not self consistent.

One possibility that there is simply no solution; that there is a deep inconsistency that does not allow a propagator to exist when the one-loop corrections are canceled by the counterterms. While this possibility cannot be neglected, it would be surprising, for the reasons outlined in the introduction. We therefore conclude that some other corrections must become relevant at low momentum. The consequence is that there must be a crossover momentum k_0 below which our solution (36) breaks down.

We can trace this breakdown back to the original simplifications we made for the integral. We found that the leading contribution to Σ_1 was

$$\Sigma_1(k, \tau_1, \tau_2) \simeq i\lambda^2 a^4(\tau_1) a^4(\tau_2) \int d^3 k_1 \int d^3 k_2 F(k_1, \tau_1, \tau_2) F(k_2, \tau_1, \tau_2) G^{R(0)}(k - k_1 - k_2, \tau_1, \tau_2) \quad (37)$$

We assumed that the main contribution came from the region of integration where the momenta k_1, k_2 were very small, and in particular much smaller than the external momentum k . Under this assumption, the integral was found to be proportional to $[c(k, \tau_1, \tau_2)]^2$ where $c(k, \tau_1, \tau_2)$ was the integral of $F(k_1, \tau_1, \tau_2)$ up to momentum k . But for small external momenta k , the last factor necessarily goes to zero. It therefore cannot be dominant for small external momenta. Once again we are led to the existence of a crossover momentum k_0 below which the solution (36) is not valid.

On the other hand, for larger momenta (k above k_0), our resummation in the previous section is still valid. We then expect the form of the propagator at these momenta to be of the form (36).

For external momenta which are smaller than k_0 , we can approximate the integral for Σ_1 by its value at $k = 0$:

$$\Sigma_1(k, \tau_1, \tau_2) \simeq i\lambda^2 a^4(\tau_1) a^4(\tau_2) \int_{k_1, k_2=0}^{\infty} d^3 k_1 d^3 k_2 F(k_1, \tau_1, \tau_2) F(k_2, \tau_1, \tau_2) G^R(-k_1 - k_2, \tau_1, \tau_2) \quad (38)$$

This is independent of k , and hence produces a correction that is dominant for low momenta. These corrections will therefore produce an effect similar to a dynamical mass, and in particular, they will cut off the infrared divergences. The time dependence is more complicated than a simple mass insertion, and so we therefore make the ansatz

$$F(k, \tau_1, \tau_2) = \begin{cases} \frac{1}{2}(F_{m_1^2(k)}(k, \tau_1, \tau_2) + F_{-m_1^2(k)}(k, \tau_1, \tau_2)) & k > k_0 \\ f(\tau_1, \tau_2) \frac{H^2}{2k^3} (k^2)^{\epsilon'} & k < k_0 \end{cases} \quad (39)$$

where $\frac{m_1^2}{3H^2} = \epsilon$, $\frac{m_2^2}{3H^2} = \epsilon'$, and we have parametrized the time dependence at low momenta by the function f .

We now can attempt to find the scaling of the various parameters of our ansatz with λ . Comparing the equations (23), which contains one integral over λF , and (38), which contains two integrals over λF , we expect $m_2^2 \sim (m_1^2)^2$, and thus that m_2^2 is parametrically smaller than m_1^2 . In turn, this means that the integrals in these equations are dominated by the low momenta regions $k < k_0$. We then find

$$m_1^2(k) \propto \lambda(k_0^2)^{-\epsilon} \left(\frac{1}{2\epsilon'}\right) \quad m_2^2 \sim \lambda^2(k_0^2)^{-2\epsilon} \left(\frac{1}{\epsilon'^2}\right) \quad (40)$$

Since ϵ' is parametrically small in λ , we can set to leading order $(k_0^2)^{-2\epsilon} = 1$. We then find $m_2^2 \sim \lambda^{2/3}$ and $m_1^2 \sim \lambda^{1/3}$.

Our conclusion for the low momentum modes is then that their propagation is modified in a manner similar to a dynamical mass. This dynamical mass has a scaling proportional to $\lambda^{1/3}$ and cuts off the infrared divergences.

VII. DISCUSSION AND CONCLUSION

We have discussed how a quartic interaction would modify the propagator of a massless field in de Sitter space. We have found a complicated result; there is a crossover momentum scale with very different behavior of the propagator above and below the crossover. For long wavelengths, the modes behave as if they develop a dynamical mass. For shorter wavelengths, surprisingly, the propagator is the sum of a massive propagator and a propagator for a tachyonic field. Our main result is encapsulated in equation (39), which is our result for the Keldysh propagator once interactions are included.

The apparently surprising behavior at shorter momenta has a natural explanation from the masslessness of the scalar. Consider the behavior of the free Keldysh propagator for $\tau_1 = \tau_2 = \tau$ with $k\tau$ large. The free propagator has the expansion

$$F_{m^2}^{(0)}(k, \tau_1, \tau_2) = \frac{H^2 \tau^2}{2k} \left[1 + \frac{1}{k^2 \tau^2} + \frac{m^2}{2k^2 H^2 \tau^2} + \dots \right] \quad (41)$$

We can therefore define the mass of the particle by looking at the deformation away from the massless propagator i.e.

$$m^2 = \lim_{k \rightarrow \infty} 4k^3 [F(k, \tau_1, \tau_2) - F_{m^2=0}^{(0)}(k, \tau_1, \tau_2)] \quad (42)$$

For the expression (36), we find that the mass is zero, thereby justifying calling this a massless scalar. At this order in the loop expansion, the high momentum modes in our calculation do not obtain a mass correction. Note that the tachyonic piece was necessary for this to happen.

The behavior for longer wavelengths is more in line with expectations. We have found that momenta with longer wavelengths behave as if they have a mass, which is similar to results in the literature. Quantitatively, however, our results are different; we have found a dynamical mass which scales as $\lambda^{1/3}$ rather than the usual scaling of $\lambda^{1/4}$ found by previous authors. This is because the mass in our case is generated at two loops rather than one-loop.

Further corrections to the propagator and other correlation functions can be calculated in perturbation theory. Infrared divergences are now absent and so the perturbation expansion should make sense. We note that any loop involving an F propagator is enhanced by a factor $\frac{1}{m^2} \sim \lambda^{-2/3}$. This indicates that the perturbation expansion is in powers of $\lambda^{1/3}$ rather than λ .

There are several open issues that still need to be addressed. In particular, we have argued for a crossover momentum, but we have not found its magnitude (even as a scaling in λ). A calculation to next order in perturbation theory might shed light on this issue. We shall leave this and other questions for future work.

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